Unit 6: Introduction to linear regression

1. Introduction to regression

GOVT 3990 - Spring 2020

Cornell University

Outline

Announcements

- Projects

Outline

Modeling numerical variables

- So far we have worked with single numerical and categorical variables, and explored relationships between numerical and categorical, and two categorical variables.
- In this unit we will learn to quantify the relationship between two numerical variables, as well as modeling numerical response variables using a numerical or categorical explanatory variable.
- In the next unit we'll learn to model numerical variables using many explanatory variables at once.

Outline

1. Housekeeping

2. Modeling numerical variables

3. Main ideas

    1. Correlation coefficient describes the strength and direction of the linear association between two numerical variables
    2. Least squares line minimizes squared residuals
    3. Interpreting the least squares line
    4. Predict, but don't extrapolate

4. Summary

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Guessing the correlation

Your turn

Which of the following is the best guess for the correlation between annual murders per million and percentage living in poverty?

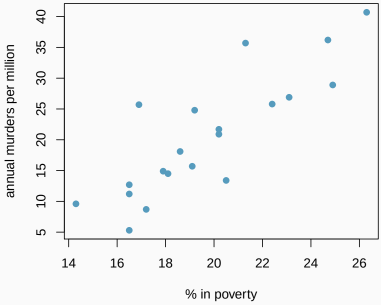
(a) -1.52

(b) -0.63

(c) -0.12

(d) 0.02

(e) 0.84



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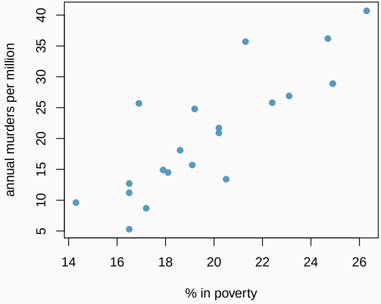


Guessing the correlation

Your turn

Which of the following is the best guess for the correlation between annual murders per million and population size?

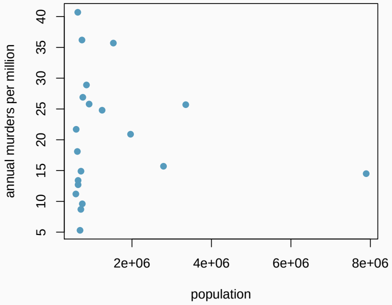
(a) -0.97

(b) -0.61

(c) -0.06

(d) 0.55

(e) 0.97



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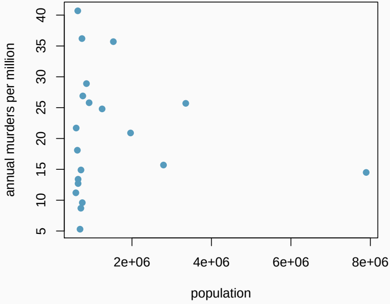
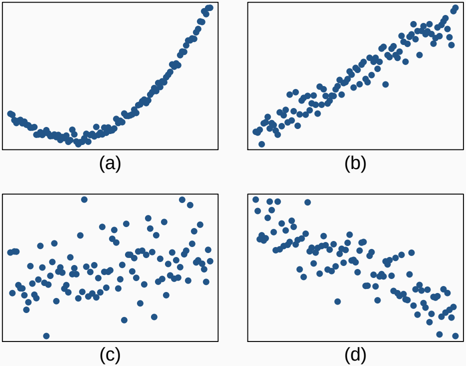


Your turn

Which of the following is has the strongest correlation, i.e. correlation coefficient closest to +1 or -1?



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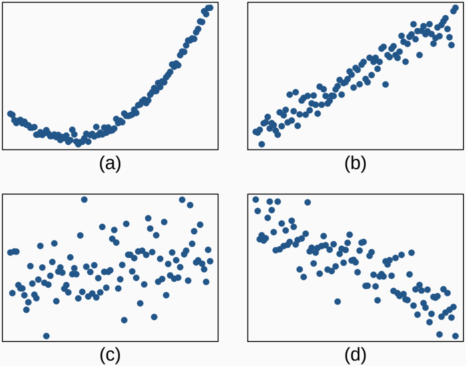

(b) $\rightarrow$ correlation means linear association

Play the game!

http://guessthecorrelation.com/

Spurious correlations

Remember: correlation does not always imply causation!

http://www.tylervigen.com/

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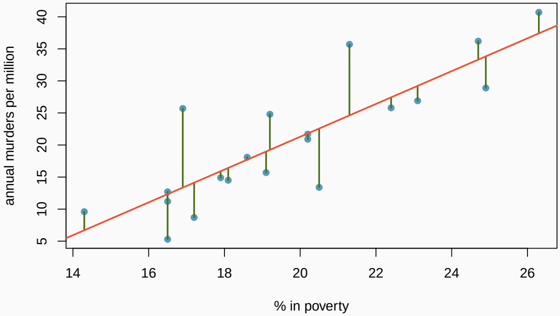
(2) Least squares line minimizes squared residuals

- Residuals are the leftovers from the model fit, and calculated as the difference between the observed and predicted y: $e_i = y_i - \hat{y}_i$
- The least squares line minimizes squared residuals:
  - Population data: $\hat{y} = \beta_0 + \beta_1 x$
  - Sample data: $\hat{y} = b_0 + b_1 x$



Outline

(3) Interpreting the last squares line

- *Slope:* For each unit increase in x, y is expected to behigher/lower on average by the slope.

$$b_1 = \frac{s_y}{s_x} R$$

- *Intercept:* When $x = 0$, y is expected to equal the intercept.

$$b_0 = \bar{y} - b_1 \bar{x}$$

  - The calculation of the intercept uses the fact the a regression line **always** passes through $(\bar{x}, \bar{y})$.

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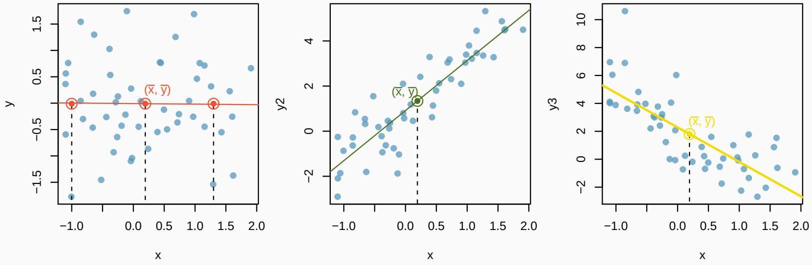
- If there is no relationship between x and y ($b_1 = 0$), the best guess for $\hat{y}$ for any value of x is $\bar{y}$.
- Even when there is a relationship between x and y ($b_1 \neq 0$), the best guess for $\hat{y}$ when $x = \bar{x}$ is still $\bar{y}$.



Application exercise: 6.1 Linear model

Your turn

What is the interpretation of the slope?

$$\widehat{murders} = -29.91 + 2.56\ poverty$$

(a) Each additional percentage in those living in poverty increases number of annual murders per million by 2.56.

(b) For each percentage increase in those living in poverty, the number of annual murders per million is expected to be higher by 2.56 on average.

(c) For each percentage increase in those living in poverty, the number of annual murders per million is expected to be lower by 29.91 on average.

(d) For each percentage increase annual murders per million, the percentage of those living in poverty is expected to be higher by 2.56 on average.

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(a) Each additional percentage in those living in poverty increases number of annual murders per million by 2.56.

(b) *For each percentage increase in those living in poverty, the number of annual murders per million is expected to be higher by 2.56 on average.*

(c) For each percentage increase in those living in poverty, the number of annual murders per million is expected to be lower by 29.91 on average.

(d) For each percentage increase annual murders per million, the percentage of those living in poverty is expected to be higher by 2.56 on average.

Outline

Your turn

Suppose you want to predict annual murder count (per million) for a series of districts that were not included in the dataset. For which of the following districts would you be most comfortable with your prediction?

A district where % in poverty =

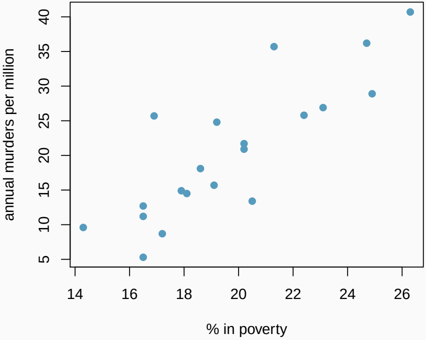
(a) 5%

(b) 15%

(c) 20%

(d) 26%

(e) 40%



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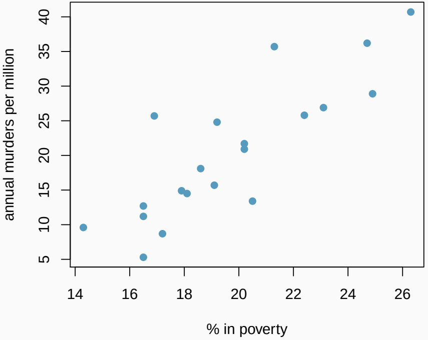
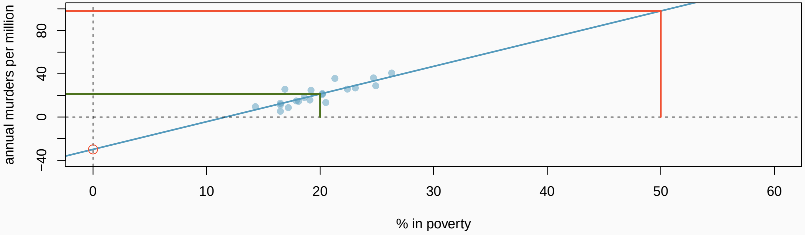


A note about the intercept

Sometimes the intercept might be an extrapolation: useful for adjusting the height of the line, but meaningless in the context of the data.



Calculating predicted values

By hand: $\widehat{murder} = -29.91 + 2.56\ poverty$

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In R:

```
# load data
murder <- read.csv("https:.../06_unit6/deck1/data/murder.csv")
# fit model
m_mur_pov <- lm(annual_murders_per_mil ~ perc_pov, data = murder)
# create new data
newdata <- data.frame(perc_pov = 20)
# predict
predict(m_mur_pov, newdata)
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Summary of main ideas

1. ??
2. ??
3. ??
4. ??