Unit 2: Probability and distributions

1. Probability and conditional probability

GOVT 3990 - Spring 2017

Cornell University

Outline

1. Main ideas
    1. Disjoint and independent do not mean the same thing
    2. Application of the addition rule depends on disjointness of events
    3. Bayes' theorem works for all types of events

2. Summary

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1. Disjoint and independent do not mean the same thing

- *Disjoint (mutually exclusive) events* cannot happen at the same time
  - A voter cannot register as a Democrat and a Republican at the same time
  - But they might be a Republican and a Moderate at the same time – *non-disjoint events*
  - For disjoint A and B: $P(A\ and\ B) = 0$

1. Disjoint and independent do not mean the same thing

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  - A voter cannot register as a Democrat and a Republican at the same time
  - But they might be a Republican and a Moderate at the same time – *non-disjoint events*
  - For disjoint A and B: $P(A \text{ } and \text{ } B) = 0$
- If A and B are *independent events*, having information on A does not tell us anything about B (and vice versa)
  - If A and B are independent:
    - $P(A \mid B) = P(A)$
    - $P(A \text{ } and \text{ } B) = P(A) \times P(B)$

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2. Application of the addition rule depends on disjointness of events

- *General addition rule:* P(A or B) = P(A) + P(B) - P(A and B)
- A or B = either A or B or both

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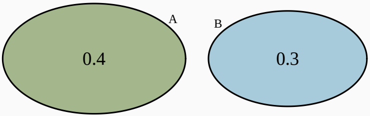
= 0.4 + 0.3 - 0 = 0.7



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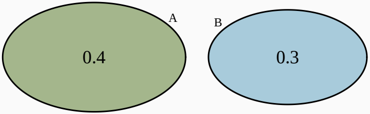
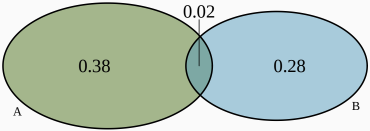


non-disjoint events:

P(A or B)

= P(A) + P(B) - P(A and B)

= 0.4 + 0.3 - 0.02 = 0.68



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  = P(A | B) × P(B)
  = *0* × P(B) = 0

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- P(A and B)
  = P(A | B) $\times$ P(B)
  = *P(A)* $\times$ P(B)

Application exercise: 2.1 Probability and conditional probability

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Summary of main ideas

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